PHYC 581: High Energy Astrophysics
Fall 2014

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Formation of a Disk:

Once matter flows across the Lagrange point L_1, it will sweep past the compact object in its Roche lobe. The plasma at L_1 has an angular momentum with respect to the primary star. It therefore forms an orbiting ring about the accretor due to dissipation of energy.

The velocity of mass flowing across the point L_1 has both parallel and perpendicular components ($v_\parallel$ and $v_\perp$ respectively). They are given by:

$v_\perp \sim b_1 \omega$, $v_\parallel \sim c_s$

The parallel component is due to thermal random motion. For typical stellar envelope temperatures $T \lesssim 10^5$ K, we have:

$v_\parallel \sim 10 \ km s^{-1}$

After using the Kepler's third law, we find:

$$v_\perp \sim 100 \left(\frac{M}{M_\odot}\right)^{\frac{1}{3}} (1+q)^{\frac{1}{3}} \left(\frac{P_{orb}}{1\,\mathrm{day}}\right)^{-\frac{1}{3}} \mathrm{km\,s^{-1}}$$

Here $\omega = \frac{2\pi}{P_{orb}}$. Within the primary's Roche lobe, the infalling mass is controlled by M_1 potential. As a result of energy dissipation, the orbit circularizes at a radius R_{circ} where the Keplerian angular momentum is equal to the initial angular momentum of the mass at L_1. Thus:

$$v_\phi(R_{circ}) = \left(\frac{GM_1}{R_{circ}}\right)^{\frac{1}{2}}$$

$$R_{circ}\, v_\phi(R_{circ}) = b_1^2 \omega$$

Together with the Kepler's law, this results in:

$$\frac{R_{circ}}{a} = (1+q)\left(\frac{b_1}{a}\right)^4 = (1+q)\left[0.500 - 0.227 \log q\right]^4$$

For $q=1$, we find $b_1 = 0.500a$, ~~$R_1 \approx b_1$~~, and $R_{circ} = 0.125a$.

It is seen that $R_{circ} = 0.33 R_1$ in this case. Actually, this result is generally true as $\frac{R_{circ}}{R_1} \sim \frac{1}{3} - \frac{1}{2}$ in compact binary systems. This ensures that the infalling mass will stay in the Roche lobe of the primary.

One has to also compare R_{circ} with the radius of the primary R_*. If $R_{circ} > R_*$, then a disk will be indeed formed. Otherwise, the accreted mass will fall onto the compact object. It can be seen that for any realistic binary parameters, we have $R_* < R_{circ}$. For example, for $P_{orb} \sim 1\,h$ and $q=1$, we find $R_{circ} \sim 3.5 \times 10^9$ cm. For a compact primary, the radius R_* cannot be larger than that for a white dwarf, and hence $R_* < 10^9$ cm. This implies that the gas flowing from L_1 toward M_1 misses the primary entirely and settles into a ring-like orbit that has ___ radius $\sim R_{circ}$. After this happens, viscosity enters the game and disperses the material to form a differentially rotating disk. We will discuss the theory of accretion disk and their physical properties in detail in the next lectures. This is an important topic because in order to fully understand a high energy source, we must have a viable theory of matter rotating on Keplerian orbits about a central accretor.